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EXPERIMENTS

ON THE

TRANSVERSE STRENGTH AND OTHER PROPERTIES

OF

MALLEABLE IRON,

WITH

REFERENCE TO ITS USES FOR RAILWAY BARS

AND A

REPORT

FOUNDED ON THE SAME

ADDRESSED TO THE DIRECTORS OF THE LONDON AND BIRMINGHAM RAILWAY COMPANY.

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PREFACE.

In order to render some remarks and observations in the following pages intelligible to the general reader, it will be necessary to state a few particulars relative to the circumstances which gave rise to the experiments, and to the appearance of them in their present form.

The Board of Directors of the London and Birmingham Rail-way Company, desirous of carrying on the great work in which they are engaged on the most scientific principles; and, if possible, to avoid the enormous cost of repairs which has attended some large works of a similar description, offered, by public advertisement,

a prize of one hundred guineas "for the most improved construction of Rail-way Bars, Chairs, and Pedestals, and for the best manner of affixing and connecting the Rail, Chair and Block to each other, so as to avoid the defects which are felt more or less on all Rail-ways hitherto constructed;" stating, that their object was to obtain, with reference to the great momentum of the masses to be moved by locomotive Steam-Engines on the Railway,

1. "The strongest and most economical form of Rail.

2. "The best construction of Chair.

3. "The best mode of connecting the Rail and Chair; and also the latter to the Stone Blocks or Wooden Sleepers. And that the Railway Bars were not to weigh less than fifty pounds per single lineal yard."

In consequence of this advertisement, a number of plans, models, and descriptions were

deposited with the Company within the time limited by the advertisement; and others were received afterwards, which, although not entitled to the prize, were still eligible to be considered with reference to their adoption for trial. On the 24th of December last, a resolution was passed at a meeting of the Directors, appointing J. U. Rastrick, Esq. of Birmingham, N. Wood, Esq. of Newcastle, Civil Engineers, and myself, to examine and report upon the same, with a view to awarding the prize; and, at the same time, we were requested to recommend to the Directors such plans, whether entitled to the prize or not, as might be considered deserving of a trial. We met accordingly in London; and, after a long and careful examination of the several plans, drawings, and written descriptions, recommended those we thought entitled to the prize, which was awarded by the Directors accordingly. But that part of our instructions which required us to recommend one or more rails for trial, we were unable to fulfil to our satisfaction, principally for want of data to determine which of the proposed rails would be strongest and

stiffest under the passing load, and whether permanently fixing the rail to the chair, for which there were several plans, would be safe in practice. No experiments on malleable iron having ever been made bearing on these points, it was considered better to leave the question unanswered, than to recommend, on no better ground than mere opinion, an expensive trial, which might ultimately prove a failure.

Seeing, however, how desirable it was that such data should be obtained, I proposed to the Directors to undertake a course of experiments, which should be conducted on a scale adequate to the importance of the subject, provided my Lords Commissioners of the Admiralty would allow me the conveniences His Majesty's Dockyard at Woolwich afforded, (which I had every reason to hope they would do, from the liberality I had so frequently experienced from that Board on similar occasions,) and that the Directors would supply such instruments, material and workmanship, as might be required for the purpose.

The Admiralty, as I had anticipated, immediately granted my request; and, at a public meeting of the Proprietors, held at Birmingham, a resolution was passed embodying my proposition. I accordingly commenced, and continued my experiments, till I had elicited such facts as I thought necessary; and having arranged them, as in the following pages, I delivered the results, with a report founded upon them, to the Secretary of the London Committee, to lay them before the Board; which being done, the Directors were pleased to express their high approbation of my labours, and their wish that the results should be made public. I have been, therefore, induced to print them in their present form, introducing only such foot notes as seemed to me necessary to render the subject intelligible to the general reader. I have given also, in addition, the solution of one or two equations, which, to avoid embarrassing the report, had been suppressed, the results only having been stated.

Such are the circumstances under which the following pages have been submitted to the

press; and they will serve to account for the form in which the subjects are arranged, which would probably have been different, if the publication in a separate work had been anticipated in the beginning. I have no doubt, however, if the facts elicited be found useful, the form and arrangement will be considered matters of secondary consideration.

EXPERIMENTS

ON THE

TRANSVERSE STRENGTH, &c. OF MALLEABLE IRON.

Preliminary Remarks.

It is only since the very general adoption of Rail-ways in this country, that malleable iron has been employed to any extent to resist a transverse strain, and writers who have undertaken experiments to investigate the strength of materials, have hitherto passed over those inquiries which relate to the transverse strength of this metal.* The extraordinary extent, how-

* Some few experiments on the transverse strength of malleable iron have certainly been made. I have given three in my Essay on the Strength of Materials. Mr. Hodgkinson has also glanced at this subject in his valuable paper of

ever, to which malleable iron is now applied to resist transversely a passing load, renders it highly essential that this resistance, and its other properties, should be fully investigated; for it is obvious, that every additional weight of metal, beyond that which is requisite for perfect safety, is not only uselessly, but injuriously employed,—it being generally admitted that bars beyond a certain weight cannot be so well manufactured as those of less dimensions; and it is no less certain, that by a proper disposition of the metal in the sectional area of the bar, (which depends on the data in question,) a greater strength may be obtained with a given weight of iron, than with a greater weight injudiciously disposed. Under these impressions, the following experiments have been undertaken, and to these inquiries only they

Experiments on Cast Iron, published in the Memoirs of the Manchester Philosophical Society, and M. Duleau has treated of the subject in his "Essai Theorique et Experimental," &c.; but those points of greatest importance connected with the application of this metal to the purposes of Railways have never formed the subject of inquiry.

have been directed; and I am not without hope that on those points they may be found useful.

Before, however, proceeding to these experimental researches, there is one subject, rather of investigation than of experiment, on which I have thought it necessary to bestow some attention, it being one on which the opinions of practical men are much divided;—this is, the comparative advantages and disadvantages of what is called the Fish-bellied Rail, and that with parallel edges.

Examination of the Properties, Curvature, and Resistance, of the Fish-bellied Rail.

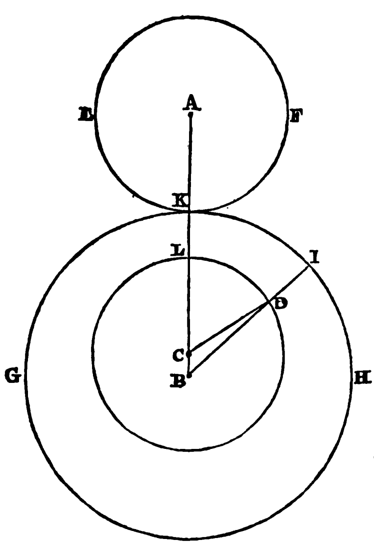
It is well known, both as a theoretical and mechanical fact, that if a beam be fixed with one end in a wall, or other immovable mass, to bear a weight suspended at the other end, the longitudinal section of such a bar (its breadth being uniform) should be a parabola; because, with that figure, every part of it will be strong in proportion

to its strain, and thus one-third of the material may be saved. This form of construction is frequently adopted in the case of cast-iron beams in buildings, and with great advantage, as thereby one-third of the material is saved, while the strength is preserved, and the walls of the building relieved from a great unnecessary weight.

This seems to have led to a somewhat similar principle of construction in what is called the Fish-bellied Rail; and the question here is, with what advantage? In the first place, it is to be remarked that the figure, which theory requires in this case, is not, as in the preceding, a parabola; for, as in the transit of the locomotive, every part of the bar has, in succession, to bear the weight; and as the strain on any part of a beam supported at each end, and loaded in any part of its length, is as the rectangle of the two parts,—the strength being as the square of the depth,—it follows that the square of the depth ought to be every where proportional to the rectangle of the two parts, which is the known property of a semi-ellipse. The bar, therefore, in theory, ought to be a semi-ellipse, having its

length equal to the transverse diameter, and the depth of the beam for its semi-conjugate, and there can be no doubt, that such a figure would be, to all intents and purposes, as strong in its ultimate resistance as a rectangular beam.

But it is difficult to obtain this figure correctly in malleable iron, and many of what are called Fish-bellied Rails are but bad approximations to it, although others differ from it but slightly. The following is the general mode of manufacture.



EF is the section of an iron roll; GH the section of another. This latter being hung on a false centre C, is turned down, leaving a groove of varying depth as shown in the figure. The cylinder GH being now again placed on its proper centre B, the bars are introduced between the two rolls at KL; and as the iron

passes through, it acquires the variable depth shown in the lower roll. The inner circle, or bottom of the groove, is generally one foot in diameter, and the upper three feet in circumference; consequently, the figure is completed in a length of three feet, and there are commonly five such lengths in a bar. The computation of the ordinates to the curve thus formed is by no means difficult; for, calling the radius of the cylinder $CD = r$, and the distance of the centres $BC = d$ and x any angle LCD, we find the ordinate.

$$ID = BI - \sqrt{(r^2 + d^2 - 2rd \cos x)}.$$

And by this formula the ordinates of the curves have been computed for two different fish-bellied rails; the extreme depth in both being five inches, but the lesser depth in one three inches, and in the other three and three-quarter inches, the latter being that proposed by Mr. Stephenson for the London and Birmingham Rail-way. The ordinates are taken for each 10°, or for every inch of the half-length, and in the last column are given the ordinates of the true ellipse.

TABLE OF ORDINATES.

ABSCISSES.			Ordinates in Fish-bellied Rail. Greatest Depth 5 In. Least ... ditto 3 „	Ordinates in Mr. Stephenson's Rail.	Ordinates in the Ellipse.
Deg.		Inch.			
0	=	0	3·00	3·75	0
10	or	1	3·01	3·76	1·64
20	..	2	3·05	3·78	2·29
30	..	3	3·12	3·82	2·76
40	..	4	3·21	3·88	3·14
50	..	5	3·31	3·95	3·46
60	..	6	3·44	4·04	3·72
70	..	7	3·59	4·14	3·96
80	..	8	3·75	4·23	4·16
90	..	9	3·92	4·34	4·33
100	..	10	4·09	4·45	4·48
110	..	11	4·27	4·55	4·61
120	..	12	4·43	4·66	4·71
130	..	13	4·59	4·75	4·80
140	..	14	4·72	4·84	4·87
150	..	15	4·84	4·91	4·93
160	..	16	4·93	4·95	4·97
170	..	17	4·98	4·99	4·99
180	..	18	5·00	5·00	5·00

We see by this table, (although it is impossible, with any proportions or degrees of eccentricity, to work out a true ellipse by this method,) that we may approximate towards it sufficiently near for practical purpose, as Mr. Stephenson has done; while, on the other hand, without due precaution, we may so far deviate from it as to

render the bar dangerously weak in the middle of its half-length.

As far as relates to ultimate strength, there can be no doubt Mr. Stephenson's rail is equal to that of an elliptic rail, and consequently to that of a rectangular rail of the same depth; but there is still an important defect in all elliptical bars, viz. that although this form gives a uniform strength throughout, it is by no means so stiff as a rectangular bar of a uniform depth, equal to that of the middle of the curved bar, and it is the stiffness rather than the strength that is of importance; for the dimensions of the rail must so far exceed those which are barely *strong enough*, as to put the consideration of ultimate strength quite out of the question. The object, therefore, with a given quantity of metal, is to obtain the form least affected by deflection; and unfortunately the elliptical bar, although equally as strong as the rectangular bar of the same depth, as far as regards its ultimate resistance, is much less stiff. This will appear from the following investigation:—

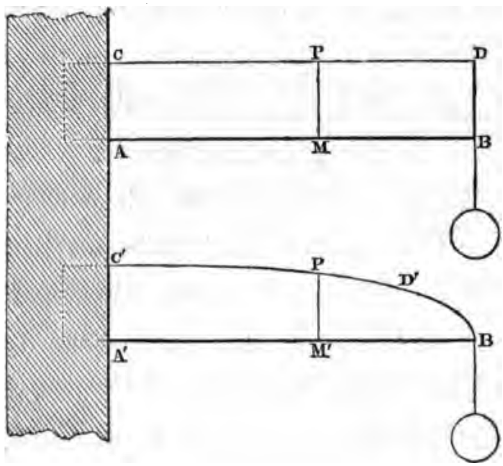


Fig. 1.

Fig. 2.

The deflections which beams sustain when supported at the ends and loaded in the middle, is the same, as the ends would be deflected, if the beams were sustained in the middle, and equally loaded at the ends, each with half the weight; and the *law* of deflection is the same in the latter case, as when the beam is fixed in a wall and loaded at its end, although the *amount* is greater. At present, however, our inquiry is not the actual, but the relative deflection in two beams, one elliptical, and the other rectangular, of the same length, and of the same extreme depth—the breadth and load being also

equal in each. It is quite sufficient, therefore, to consider the corresponding effects on two half-beams, each fixed in an immovable mass, as represented in the preceding figures.

Now, in the first place, the elementary deflection at C is the same in both beams, because the lengths and loads are the same, and the depths at CA equal; but the whole deflection at any other point P, will be directly as MB^2, and inversely as MP^3. If, therefore, we call $MB=x$, and $MP=y$, the sum of all the deflections in the two beams will be $\int \frac{x^2}{y^3} . \, dx \, \triangle$, $\triangle$ being the sine of deflection at C. But in fig. 1, y is constant and equal to d (the depth,) while in the latter,

$$y = \frac{d}{l} \sqrt{(2\,lx - x^2)}$$

l being the semi-transverse or length, and x any variable distance.

The whole deflections, therefore, in the two cases, are,

Fig. 1. Deflection $= \int \frac{x^2 \, dx}{d^3} \triangle =$ (when $x=l$)

$$\frac{1}{3} \frac{l^3}{d^3} \triangle$$

And in Fig. 2 :—

$$\text{Deflection} = \int \frac{x^2\, dx\ \triangle}{\frac{d^3}{l^3}\,(2\ lx - x^2)^{\frac{3}{2}}} = (\text{when } x = l)$$

$$\cdot 41\ \frac{l^3}{d^3}\ \triangle$$

The deflections, therefore, in the two cases are, with the same weights, as 33 to 41,* or nearly as 3 to 4, a result fully borne out by subsequent experiment. It is to be observed also, that this investigation applies only to the deflection when the weight is in the middle of the bar, and that it would be much greater in comparison with the parallel rail towards the middle of its half-length.

This want of stiffness is, I should imagine, but badly compensated by the trifling saving of metal thus effected; for I find that an addition of little more than four pounds per yard, would convert this rail into a rectangular one of the

* Experiments have been made from which it has appeared, that the fish-bellied rail was stiffer than the parallel rail, which is certainly possible, if the parallel rail be of inferior metal or of injudicious figure; but it is mechanically impossible if the parallel bar be made of the figure here assumed.

same depth, which would have one-third more stiffness at its middle point, and probably one-half more, a little beyond the middle of the half-lengths. I am aware, objections are made to rectangular bars having so much depth of bearing in their chairs, and this may be a practical defect, on which I shall offer no opinion; at all events, it is well to estimate properly both evils, and then to choose the least.*

Having thus satisfied myself on the nature of the fish-bellied rail, I proceeded with my experimental inquiries, which I have divided into the following sections:—

1. To determine the extension of an iron bar of given area, under different degrees of tension; and hence the force with which the same bar will contract with a given reduction of temperature.

2. The comparative resistance of malleable iron to extension and compression, and thereby the position of the neutral axis.

3. The figure of the area of section, which

* It will be seen in a subsequent page, that by introducing what is called a lower web, that weight for weight, a parallel rail may be made as strong as the fish-bellied, with only an additional depth in the chair of three-quarters of an inch.

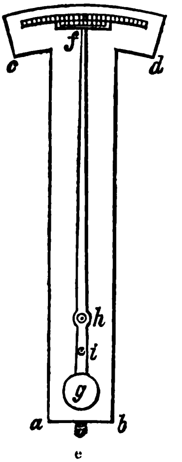
gives the greatest strength with the same quantity of metal.

4. The strains which bars of given sections are capable of sustaining without injury to their elastic power.

Experiments to determine the quantity which iron extends under different degrees of tension.

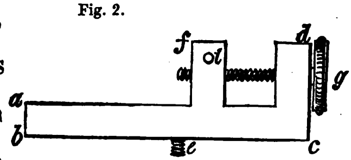
With a view to this inquiry, an instrument was made as in the annexed sketch.—*abcd*, is a piece of brass, about one-fifth of an inch thick, having an arc at top, divided into tenths of inches; *hfg* is a hand, with a vernier, turning freely on a centre *h;* and *i* is a steel pin, about half-an-inch long, projecting perpendicularly forward; the distances *fh* to *hi* being as 10 to 1. *e* is a small end with

Fig. 1.

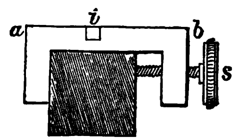


a screw, for the purpose described below. *abcd,* is another piece of brass, having a screw *e; f* is a piece working in a dove-tail, adjustable for position by the screw *g*, and *i* is another steel pin projecting forward. *a b*

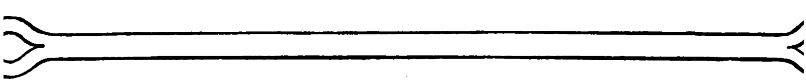
Fig. 2.



is an iron saddle-piece, with a set screw *s;* and at *i* a hole is tapped to receive the screw *e*, fig. 2; and another saddle piece, exactly like this, is made to receive the screw *e*, of fig. 1.



The iron bars intended to be experimented on were made of the annexed form, about ten feet



in length; these, by proper bolts and sheckles, were fixed at *a* and *b* in the proving machine;* the

* The Lords Commissioners of the Admiralty having been pleased to allow me any facilities His Majesty's Dock-yard at Woolwich afforded for conducting these experiments on a proper scale, the proving machine here referred to is an hydrostatic press, constructed by Messrs. Bramah's, principally for the purpose of testing or proving the iron cables, before

two saddle pieces were then fixed on at the exact distance of 100 inches; the instruments, fig. 1 and 2, screwed into their respective saddle pieces, and a light deal rod hung, by means of two small holes formed in it, (also at the distance of 100 inches,) upon the two pins *i i;* and then by means of the set-screw, fig. 2, the vernier of fig. 1. was adjusted exactly to zero. The pump of the hydraulic press was now put in action, and after one, two, or more tons pressure were on, according to the size of the bar, and every thing brought well to its bearing, the hand was again adjusted to zero, after which the index was read for every additional ton. Here it will be seen, that whatever the bar stretched between the two instruments, the lower pin of fig. 1. was drawn forward, and the index-end thrown back ten times that amount, consequently to ten times the actual amount of the quantity stretched.

It has been observed, that after one, two, or more tons strain were applied to bring every

they are issued for service. It is an excellent machine of its kind, is capable of bearing a strain of 100 tons, and is very sensible to a difference of strain of 1-8th of a ton.

thing well to its bearing, the index was adjusted to zero, and its reading afterwards carefully registered as each additional ton was added. The strain during the experiment was repeatedly let off, and the index was found to return to zero, till the strain amounted to about nine or ten tons per inch, when the stretching became greater for each ton, and the bar did not any longer regain its original length when the strain was removed, its elasticity with this tension being obviously injured.

These experiments required more attendance than it was possible for one person to give; the adjustment of the weights, the reading and registering the index, required each the undivided attention of one individual; the pumping also required to be watched with care. And I have great pleasure in acknowledging the ready assistance I received from Messrs. Lloyd and Kingston, the Engineers of the yard; from Mr. P. W. Barlow, Civil Engineer; as also from Lieutenant Lecount, who came from Birmingham to witness and assist in the experiment.

Experiments on the Longitudinal Extension of Malleable Iron Bars, under different Degrees of direct Tension.

TABLE I.

Bar No. 1, 1 inch square. February 21st.			Bar No. 2, 1 inch square. February 21st.		
Weight in Tons.	Index Readings.	Parts of the whole Bar extended by each Ton.	Weight in Tons.	Index Readings.	Parts of the whole Bar extended by each Ton.
2	zero		2	zero	
3	·0625	·0000625	3½	·11	·0000733
4	·156	·0000935	4	·15	·0000800
5	·265	·0001090	5	·24	·0000900
6	·375	·0001100.	6	·35	·0001100
7	not observed	mean.	7	·44	·0000900
8	·562	·0000935	8	·52	·0000800
9	not observed	mean.	9	·62	·0001000
10	·750	·0000940	10	·70	·0000800
11	·875	·0001250	11	·81	·0001100
			12	1·13	Elasticity injured

Bar No. 3, 1 inch diameter. February 23d.			Bar No. 4, 1-inch diameter. February 23d.		
Weight in Tons.	Index Readings.	Parts of the whole Bar extended by each Ton.	Weight in Tons.	Index Readings.	Parts of the whole Bar extended by each Ton.
1	zero		1	zero	
2	·16	·0001600	2	·15	·0001500
3	·31	·0001500	3	·28	·0001300
4	·44	·0001300	4	·42	·0001400
5	·56	·0001200	5	·56	·0001400
6	·67	·0001100	6	·69	·0001300
7	·79	·0001200	7	·79	·0001000
8	·91	·0001200	8	·97	·0000800
9	·103	·0001200	9	·116	Elasticity destroyed

Mean extension per ton, per square inch

Bar No. 1.	·0000982
No. 2.	·0000903
No. 3.	·0001010
No. 4.	·0000976
Mean of the four . .	·0000967

TABLE II.

Bar No. 5, 2 inches square. February 28th.			Bar No. 6, 2 inches square. February 28th.			Bar No. 7, 2 inches square. March 7th.		
Weight in Tons.	Index Readings.	Parts of the whole bar extended by each 4 Tons.	Weight in Tons.	Index Readings.	Parts of the whole bar extended by each 4 Tons.	Weight in Tons.	Index Readings.	Parts of the whole bar extended by each 4 Tons.
4	zero		4	zero		4	zero	
6	·100		6	·090		6	·065	
8	·180	·000180	8	·150	·000150	8	·125	·000125
10	·240	·000140	10	·210	·000120	10	·175	·000110
12	·290	·000110	12	·250	·000100	12	·230	·000050
14	·350	·000110	14	·290	·000080	14	·280	·000050
16	·400	·000110	16	·335	·000085	16	·335	·000050
18	·450	·000110	18	·375	·000080	18	·385	·000105
20	·500	·000100	20	·410	·000075	20	·435	·000100
22	·550	·000100	22	·445	·000070	22	·480	·000095
24	600	·000100	24	·485	·000075	24	·530	·000095
26	·650	·000100	26	·525	·000080	26	·575	·000095
28	·695	·000095	28	·565	·000080	28	·625	·000095
30	·740	·000090	30	·620	·000095	30	·670	·000095
32	·790	·000095	32	·660	·000095	32	·715	·000090
34	·825	·000085	34	·730	·000110	34	·755	·000085
36	·860	·000075	36		Full elasticity.	36	805	·000090
38	·920	·000095	38			38	·850	·000095
40	1·05	·000145	40			40	·900	·000095
		Elasticity exceeded						Elasticity perfect

Mean extension per ton, per square inch, No. 5. ·0001082
No. 6. ·0000957
No. 7. ·0000841

Mean . . . . ·0000946
Mean of preceding Table . . . ·0000967

Collecting the results of these seven experiments, and reducing them all to square inches, we find that the strain which was just sufficient to balance the elasticity of the iron, was in—

Bar, No.	1.	(re-manufactured iron)	10 tons.
	2.	ditto	11 tons.
	3.	New Bolt	11 tons.
	4.	ditto	10 tons.
	5.	(re-manufactured)	9·5 tons.
	6.	ditto, from old furnace bars	8·25 tons.
	7.	New bar, by Messrs. Gordon	10 tons.

We may consider, therefore, that the elastic power of good iron is equal to about ten tons per inch, and that this force varies from ten to eight tons in indifferent and bad iron. It appears, also, (considering ·000096 as representing in round numbers $\frac{1}{10000}$th) that a bar of iron is extended one ten-thousandth part of its length by every ton of direct strain per square inch of its section; and consequently, that its elasticity will be fully excited when stretched to the amount of one-thousandth part of its length.

Remarks on the foregoing Experiments.

These results have an important bearing on the question of rail-way bars. We shall see, in the following section, how they become applicable to the investigation of the transverse strain; but, at present, I shall only speak of them as they apply to the fixing of the rail to the chair. Amongst the numerous models which the Directors did Messrs. Rastrick, Wood, and myself, the honour to submit to our inspection, for the purpose of awarding their prize, there were several in which it was intended to fix the rail permanently to the chair—a very desirable object, if it could have been safely adopted; and it was the want of data to enable us to decide on this point, which first led me to propose this course of experiments. The question is now satisfactorily answered. We have seen that, with about ten tons per inch, a bar of iron is stretched $\frac{1}{1000}$th part of its length, and its elasticity wholly excited or surpassed. Again, admitting 76° to be the extreme

range of the thermometer in this country between summer and winter, it appears, from the very accurate experiments of Professor Daniell,* that a bar of malleable iron will contract with this change $\frac{1}{2000}$th part of its length. And hence it follows, that if the rails were permanently fixed to the chair in the summer, the contraction in the winter would bring a strain of five tons per inch upon the bar, and a strain of twenty-five tons upon the chair, (the bar being supposed of five-inch section,) thereby deducting from the iron more than, or full half, its strength, and submitting the chair to a strain very likely to destroy it. Every proposition, therefore, for permanently attaching the rail to the chair is wholly inadmissible.

These remarks may also be carried still farther. If it be dangerous to attach the rail *directly* to the chair, it must be bad in practice to affix it *indirectly* by wedges, cotters, or otherwise, beyond what is absolutely essential to give it steadiness under the passing load; for it is evident, that if by these means we could prevent

* See Phil. Trans. 1831.

any motion taking place, we should fall into the same evil as by the permanent attachment; and if, as most probably will happen, we fail of entirely accomplishing this, still all the friction which is produced must be overcome by the contracting force of the iron, and be so much strength deducted from its natural resisting power.

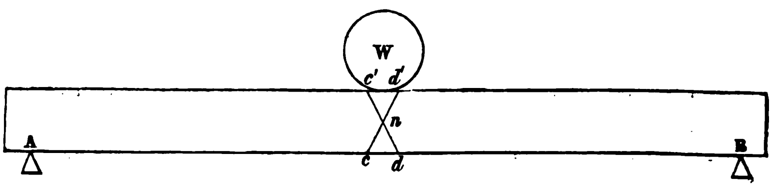
The problem, therefore, which engineers have to solve, is, "To find a mode of fixing the rail to the chair, which shall give sufficient steadiness to the former: but which, at the same time, shall produce the least possible resistance to the natural expansion and contraction of the bar."

The quantity of motion which thus takes place is certainly but small, viz. about $\frac{1}{11}$th of an inch between summer and winter, with a fifteen-foot bar; but the force of contraction is great, amounting to five tons per sectional inch for the annual extremes, and frequently to not less than two and a half tons between the noon and night of our summer season, while the whole power of iron within the limits of its elasticity does not exceed nine or ten tons.

This is an important consideration, and for want of attention to it, or rather, in consequence of its amount not having been ascertained, a practice of wedging or fixing the rails has prevailed, which must necessarily have been the cause of great destruction to the bars.

I would also suggest here, as a matter deserving the attention of practical men, that as the bar must necessarily contract, it will draw from that side, which is least firmly fixed, and hence all the shortening will most probably be exhibited at one end, however slight the hold on either may be; and when it happens that the adjacent ends of two bars both yield, the space between the two is rendered double that which is necessary. To avoid this evil, one of the two middle chairs in each bar might be permanently attached to the rail, in which case the contraction must necessarily be made from each end, and the space occasioned by the shortening of the bars would then be uniform throughout, and much unnecessary and injurious concussion thus saved both to the rail and to the carriage.

Experiments to determine the comparative Resistance of Malleable Iron to Extension and Compression, and the position of the neutral axis in bars submitted to a transverse strain.



Let A B represent an iron or any other bar supported at A and B, and loaded in the middle by a weight W, which deflects it; extending the fibres between *n* and *c d*, and compressing those between *n* and *c' d'*. Now, supposing the system in equilibrio, $\frac{1}{2}$ W acting at the extremity of the $\frac{1}{2}$ length, or $\frac{1}{4}$ *l* W, is equivalent to the sum of all the resistances to extension in *n c d*, and to all those of compression in *n c' d'*, each fibre acting on a lever equal to its distance from the neutral axis *n*. Consequently, as the quantity of extension of any fibre is as its distance from the neutral axis, and the lever by which it

acts, being also as that distance, the actual resistance of a fibre at the distance, x, is as $\frac{x^2 t}{d'}$, t being the tension of the lower fibre, and d' its depth below the neutral axis; and the sum of all these resistances will be $\int \frac{t\,x^2 d\,x}{d'} = \frac{1}{3} d'^2 t$, (when $x = d'$) or for the whole depth. In the same way, c being taken to denote the compression of the upper fibre, corresponding to the tension t, the sum of all the compressions will be,

$$\frac{1}{3} d''^2 c,$$

d'' denoting the depth of compression; hence the whole sum is,

$$\frac{1}{3} d'' \; c + \frac{1}{3} d'^2 t = \frac{1}{4} \mathrm{W}\, l;$$

but $d'' c = d' t$,* the quantity of resistance being

* To prevent misapprehension, it may be proper to observe that c here, is not intended to represent the force requisite to compress a fibre the same quantity that the force t extends it; but simply, the force of the compression at c, corresponding to the tension t on the lower fibre. The equation, therefore $d''\,c = d'\,t$ is equivalent to saying that the sum of all the forces in $n'\,c'\,d$ is equal to all the forces in $n\,c\,d$; or that $a\,g = n\,a'\,g'$; a, a', denoting the areas, and g, g' the distances of the centres of gravity from n, and taking $n\,t$ to denote the force which will compress a fibre to the same extent as the force t will extend it.

equal to that of extension; this, therefore, becomes

$$\frac{1}{3}\,d''d'\,t+\frac{1}{3}\,d'^2t=\frac{1}{4}\,l\,\mathrm{W},\ \text{or}$$

$$\frac{1}{3}(d''+d')\,d'\,t=\frac{1}{4}\,l\,\mathrm{W},\ \text{or}$$

$$\frac{1}{3}\,d.\,d'\,t=\frac{1}{4}\,l\,\mathrm{W}.\,;$$

d being the whole depth, and d' the depth of tension; whence

$$d'=\frac{3\,l\,\mathrm{W}}{4\,d\,a\,b}=\text{depth of tension, and}$$

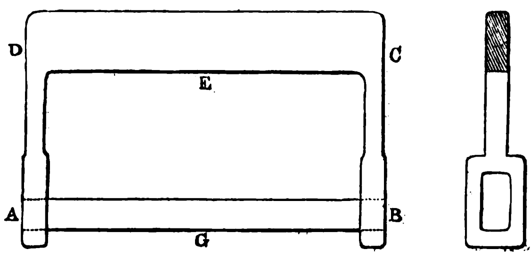
$$d-d'\ \text{the depth of compression,}$$

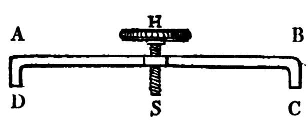
consequently, $\frac{d'}{d-d'}$, the ratio, in which the neutral axis divides the sectional area in rectangular bars.

Comparison of the Formula with Experimental Results.

In order to submit this formula to practical results, a strong iron frame was forged, of the form shown in the annexed figure; D C is thirty-six inches long, six inches broad, by two deep; the two arms two inches square, and the ends

of proportional dimensions to those represented. The other view of the arms is represented in the side figure, with an opening six inches by three, in which the bars for experiment were placed, as represented by A G B; the space between is thirty-three inches. The shackles were applied at E and G, and connected by strong iron cables to the press, the strain was then brought on and the results recorded.





In order to measure with every requisite accuracy, the deflections which the bar sustained, as different weights were applied, an instrument of the form shown in the annexed figure was neatly and accurately made in iron, having two feet, A D, B C; the centre was tapped to receive the brass screw, H S, of

twenty threads to the inch, and the head was divided into five equal parts, and by again subdividing these divisions into ten, a deflection of $\frac{1}{1000}$ of an inch might be measured with great ease.

The method of applying it was to rest its feet on the bar, and then to retain it in its place by cramps and screws. The micrometer screw was then run down till it was in contact with the bar, and the divisions read and registered, either before any strain was on, or when the first slightest strain could be estimated, as stated in the following table.

The first six experiments were made on different parts of the bars, Nos. 5, 6, and 7, without cutting them, by introducing them into the iron frame above described (having thirty-three inches clear bearing,) and straining them till the successive deflections showed a tendency to increase in amount, which was taken as a sign of the elasticity being injured; and the amount of this strain having been previously ascertained by the former experiments, they furnish the best possible data to apply to the formula for determining the position of the neutral axis.

Experiments made to ascertain the Deflections due to different Transverse Strains, and the Weight which first produces a Strain equal to the Elastic Power, and thence the position of the Neutral Axis.

TABLE III.

Part 1. Bar No. 5. Bearing 33 Inches. 2 Inches Square.

Weight in Tons.	Readings by Scale.*	Deflections for each Half Ton.
No Weight	1·96	
·875	1·92	·023
1·00	1·90	
1·50	1·90	·016
2·00	1·88	·020
2·50	1·86	·020
Weight removed.	returned to 1·96	
3·00	1·80	Elasticity injured.
Weight removed.	1·88	

Part 2. Bar No. 5. Bearing 33 Inches. 2 Inches Square.

Weight in Tons.	Readings by Scale.	Deflections for each Half Ton.
No Weight	1·95	
·750	1·92	·020
1·00	1·91	·020
1·50	1·89	·020
2·00	1·86	·030
2·50	1·84	·020
Weight removed.	returned to 1·95	
3·00	1·67	Elasticity injured.
Weight removed.	1·81	

Part 1. Bar No. 6.

Weight in Tons.	Readings by Scale.	Deflections for each Half Ton.
No Weight		
·50	1·56?	
1·0	1·50	
1·5	1·48	·020
2·0	1·45	·030
2·5	1·24	·210 Elas. injd.
3·0		

Part 2. Bar No. 6.

Weight by Tons.	Readings by Micro. Screw.	Deflections for each Half Ton.
No Weight	·025	
·50	·043	·018
1·0	·068	·025
1·5	·091	·023
2·0	·128	·037 injd.
2·25	·178	·100
2·50	·313	·185

* In the first of these experiments the deflections were measured by a scale in front of the bar, the micrometer screw not being ready.

TABLE III.—(*continued.*)

Part 1. Bar No. 7.

Weight in Tons.	Readings by Micro. Screw.	Deflections for each Half Ton.
No Weight	·031	
·50	·053	·022
1·0	·077	·024
1·5	·096	·019
2·0	·126	·030
2·5	·147	·021
3·0	·211	·064 inj^{d}.

Part 2. Bar No. 7.

Weight in Tons.	Readings in Micro. Screw.	Deflections for each Half Ton.
No Weight	·025	
·50	·056	·031
1·0	·077	·021
1·5	·098	·021
2·0	·109	·011
2·5	·137	·028 inj^{d}.
3·0	·180	

Part 3. Bar No. 7.

Weight in Tons.	Readings by Micro. Screw.	Deflections for each Half Ton.
No Weight	·075 ?	
·50	·130	
1·0	·153	·023
1·5		·023
2·0	·199	·023
2·5	·220	·021
3·0	2·90	·070 inj^{d}.

Part 2. Bar No. 7. Reversed.

Weight in Tons.	Readings by Micro. Screw.	Deflections for each Half Ton.
No Weight	·025	
·50	·054	·029
1·0	·092	·038
1·5	·153	·061
2·0	·235	·082

Elasticity clearly injured by the former experiment.

It appears from these experiments, that both parts of the Bar No. 5, (whose direct elasticity was 9·5 tons,) had their restoring power just preserved with a transverse strain of two and a half tons on a bearing length of thirty-three inches. Hence in the formula:—

$$d' = \frac{3\, lw}{4\, dat}$$

we have $l=33$, $w=2\frac{1}{2}$, $d=2$, $a=2$, $t=9{\cdot}5$, and $d'=1{\cdot}62$ inches, depth of tension.

Consequently $d''={\cdot}38$ inches, depth of compression, and the ratio of the area of compression to tension . . . . . . 1 : 4·3.

In the first part of Bar No. 6, w is not quite 2 tons, and $t=8{\cdot}5$ tons; and hence the ratio . . . . 1 : 2·7

In the second part of the same bar, ditto 1 : 2·7

In the first, second, and third parts of Bar No. 7, $w=2\frac{1}{2}$ tons, and $t=10$ tons . 1 : 3·4

As far as these experiments are authority, therefore, the neutral axis divides the sectional area of a rectangular bar in about the ratio of one to three and a half.

In the following experiments, the iron was all supplied by Messrs. Gordon, and was of the

same quality as the Bar No. 7,—its elasticity may therefore be taken as ten tons, but it was not determined by testing, as in the preceding experiments.

TABLE IV.

BAR No. 8.

Distance of bearing.	Breadth.	Depth.	Weights.	Deflections.	Deflections each ½ Ton.	REMARKS.
inches.	inches.	inches.	tons.			
33	1·9	2	·125	·034		
			·250	·046		
			·500	·060		
			1·00	missed.	·019	
			1·50	·098	·019	Mean ·024
			2·00	·120	·022	
			2·25	·134	·028	$w = 2·25$. Neutral axis 1 : 3·4
			2·50	·151	·034	
			2·75	·176	·044	Elasticity injured with 2·50 T.
BAR No. 9.						
33	1·9	2	·250	·047		
			·500	·055	·016	
			1·00	·077	·022	
			1·50	·097	·020	Mean ·021
			2·00	·123	·026	
			2·25	·132	·018	
			2·50	·145	·026	$w = 2·25$. Neutral axis 1 : 34
			2·75	·164	·038	Elasticity injured with 2·50
			3·00	·210	·092	Ditto destroyed with 3·00
BAR No. 10.						
33	1·9	2	·500	·056		
			1·00	·076	·020	
			1·50	·095	·019	Mean ·024
			2·00	·124	·029	
			2·50	·151	·027	$w = 2·5$. Neutral axis 1 : 4·2
			3·00			

Deductions from the three last Experiments, confirmed by direct Observation of the place of the Neutral Axis.

These experiments, like the former, imply, according to the formula, that the neutral axis lies at about one-fourth or one-fifth of the depth of the bar from its upper surface; but a method was adopted in these to discover, if possible, its position mechanically. With this view, a key-way, or groove, was cut in the side of the bar one inch broad, and one-tenth of an inch deep,—thus reducing the breadth to 1·9 inches. To this key-way, or groove, was fitted a steel key, which might be moved easily; and when the strain was on, the key was introduced, which it was expected would be stopped at the point where the compression commenced, and this was accordingly found to be the case in two out of the three bars, but not in the third, the fitting not being sufficiently accurate. The other two, however, showed obviously a contraction of the groove, at about half an inch from the top,

agreeing with the preceding computations. To make the results more certain, three other bars, exactly like the former, had deeper grooves cut, and the key more exactly fitted, and with these the results were as definite as could be desired. The key, as above-stated, moved smoothly and easily before the experiment; but when two tons strain were on, and the key applied, it was stopped, and stuck at a definite point. The strain being then relieved, the key fell out by its own weight; the strain was again put on, the key sticking as before; the strain being relieved, the key again fell, and so on, as often as repeated. Precisely the same happened with all the three bars. One of them was then reversed, so that the part which had been compressed was now extended, and exactly the same result followed; showing, most satisfactorily, that our former computed situation of the neutral axis was very approximative. The measurements obtained in these experiments being tension 1·6, compression ·4, giving exactly the ratio of 1 to 4 in rectangular bars. These results seem the most positive of any hitherto obtained; still, there can be little doubt this ratio varies in

iron of different qualities; but looking to the preceding experiments, it is probably always between 1 to 3, and 1 to 5.

On the Stiffness of Rectangular Iron Bars, and their Deflections under different Weights.

Although it is necessary to know the actual resisting power of bars in their ultimate state of strain, in order to determine the relative strengths of differently-shaped bars, yet the question of most practical importance is, the stiffness they exhibit when loaded with smaller weights; for we ought never to strain a bar so nearly to its full power of bearing, as to make this the immediate subject of inquiry.

The experiments recorded in the last section are applicable to this purpose, but as these are all of the same depth, it was thought more satisfactory to make a few other experiments on bars of different breadths and depths. These are given in the following page. They were performed precisely like the last, and therefore require no particular description.

Experiments on the Deflection of Malleable Iron Bars, under Different Strains.

BAR No. 11.

Distance of bearings.	Breadth.	Depth.	Weight.	Deflections.	Deflections for each half Ton.	REMARKS.
inches.	inches.	inches.	Tons.			
33	1.5	3	·125	·043		
			·500	·059		
			1·00	·074	·015	
			1·50	·083	·009	
			2·00	·095	·012	
			2·50	·101	·006	Mean ·0103
			3·00	·109	·008	
			3·50	·120	·011	
			4·00	·131	·011	
			4·50	·148	·017	$w=4\frac{1}{2}$. Neutral axis 1 : 4·9 Elasticity preserved at $4\frac{1}{2}$ tons.

BAR No. 12.

33	1·5	3	0	0		
			·50	·017		
			1·00	·037 ?		
			1·50	·052	·015	
			2·00	·061	·009	
			2·50	·064	·003	
			3·00	·078	·014	Mean ·0108
			3·50	·089	·011	
			4·00	·102	·013	
			4·50	·124	·022	$w=4\frac{1}{2}$. Neutral axis 1 : 4·9. Elasticity injured.

BAR No. 13.

33	1·5	2·5	0	·006		
			·50	·003	·024	
			1·00	·050	·020	
			1·50	·060	·010	
			2·00	·074	·014	Mean ·0173
			2·50	·093	·019	$w=3$. Neutral axis 1 : 4·9
			3·00	·110	·017	Elasticity preserved, 3 tons.
			3·50	·149		
			7·5	Bent 8	inches	

To reduce the law of deflection from these results, we may have recourse to two well-known and well established formulæ:—viz.

$$\frac{l\,w}{4\,a\,d^2}=S \text{ and } \frac{l^3\,w}{a\,d^3\,\delta}=E,$$

which are both constant quantities for the same material, w being the greatest weight the bar will bear without injuring the elasticity; consequently, when l is also the same in both, $d\ \delta$ will be also constant, a being the breadth, d the depth, and δ the deflection. That is, all rectangular bars having the same bearing, length, and loaded in their centre to the full extent of their elastic power, will be so deflected, that their deflection (δ) being multiplied by their depth (d) the product will be a constant quantity, whatever may be their breadths or other dimensions, provided their lengths are the same.

Let us see how nearly our several results agree with this condition.

In the several bars, Nos. 8, 9, 10, 11, 12, 13, multiplying the mean deflection for each half ton, by the number of half tons which excited

its whole elasticity, and this again by the depth of the bar, we find

	Depth.
No. 8, ultimate deflection	·108 × 2 = ·2160
No. 9	·094 × 2 = ·1880
No. 10	·120 × 2 = ·2400
No. 11	·0876 × 3 = ·2628
No. 12	·0918 × 3 = ·2754
No. 13	·1038 × 2½ = ·2595
	6) 1·4417
Mean......	·2403

There is rather a large discrepance in bar, No. 9; the others are as approximative to the mean as can be expected in such cases.

If we make the same trial on the three parts of bar, No. 7, we have

1st part	·116	×	2	=	2320
2d part	·105	×	2	=	·2100
3d part	·115	×	2	=	·2300
					3) 6720

Mean	·2240
Former Mean ..	·2403
	2) ·4647
General Mean...	·2323

We may therefore say, that any malleable iron bar, of 33 inches bearing, being strained to its full elasticity, will be so deflected, that its depth, multiplied by the deflection, due to 30 inches, will produce the decimal ·23; consequently $\frac{\cdot 23}{d}$ = the deflection, d being the whole depth in inches.

In this form, however, it applies only to rectangular bars. To make it general, we must estimate it from the neutral axis, which in rectangular bars, being $\frac{1}{5}$th of the depth below the upper surface, the above constant, when thus referred, becomes $\cdot 2323 \times \frac{4}{5} = \cdot 1858$. But, on the other hand, our instrument for measuring the deflection was but 30 inches long; it has therefore to be increased again in the ratio $30^2 : 33^2$, or as $10^2 : 11^2$ on this account; so that, ultimately, the formula is $d' \delta = \cdot 22$. d' denoting now the

depth of the bar below the neutral axis, and in this form it is general for parallel rails of any section whatever.

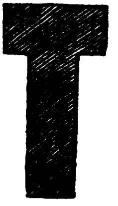
A curious circumstance was observed in these experiments, which, although it has no immediate bearing on the subject in question, it may be well to notice, and which is, I apprehend, characteristic of good malleable iron, viz. that the resistance to compression, although so much greater than the resistance to extension, is the first of the two which loses its restoring power; for if we so far increased the strain as to overcome the elastic power, the point of compression descended to nearly the middle of the depth, proving that the tensile force, although so much less, is the most tenacious; whereas I suspect, that in cast iron it is the reverse, that is, it is here the tensile power which first yields, and the consequence is a sudden fracture, and momentary destruction of the bar.

On the Sectional Figure of greatest Resistance, the Area being given.

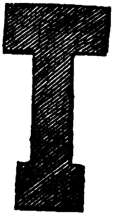
Having established the preceding data, I might now proceed directly to find, with a given sectional area, the figure of greatest resistance; but this would be of little advantage, for the form we should arrive at would be quite inapplicable to a railway, as it would require the metal to be principally collected in the lower table; whereas, in the railway bar, we must of necessity bestow a certain quantity, perhaps two-fifths of the whole, in forming the upper table on which the carriage runs; it is, therefore, only after this is provided for, that we are at liberty to dispose of the remaining part of the metal, and even in this distribution regard must still be had to practical convenience. Instead therefore, of determining, mathematically, the area of maximum resistance, the most useful plan will be to compute, directly, the resistance of such sectional figures as fall within the limits of practical application, and to select from them that which, under all considerations, is the best.

The three forms of rails which, under this restriction, will have to be considered, are the following:—

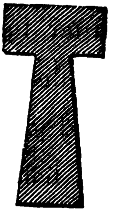
1. The plain T shaped rail, fig. 1.



2. The H, or double T, formed ra il, with a lower table, as fig. 2.



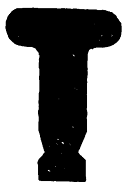
3. The Trapezoidal rail, as fig. 3.



Each of which will admit of various changes of proportions, without altering the general character of the section.

The upper and the lower tables are here represented as rectangular, with sharp edges. In

practice these are rounded off, the metal thus displaced furnishing a sort of bracket between the table and stem, or rib, as shown in fig. 4, but to treat of them in this form would introduce great intricacy into the calculation, without much affecting the results. It will therefore be sufficient to consider them as rectilinear.



I would here observe, also, that some projectors have made the upper and lower tables of equal figure, upon the distant contingency, that when the upper table has been worn down, the rail may be turned, and the lower table made the upper. But this is certainly providing without foresight; for the bottom table is the most efficient for strength, and it would be a very dangerous experiment, after one side of a bar has been submitted for many years to a high compressing force, and its substance (by the hypothesis) greatly worn, to turn the rail, and expose this worn part to a still greater strain, but tensile instead of compressive, which could not fail instantly to destroy it. Instead of this, therefore, I should certainly recommend to work

whatever metal is introduced into the lower table or web, into that form which is most efficient for present purposes, without regard to the contingency alluded to above.

That the rail is deteriorated by exposure and wear is undoubtedly true, although, perhaps, the amount is not yet well ascertained. Amongst the papers submitted to Messrs. Rastrick and Wood, with whom I was associated, we found it estimated at the rate of $\frac{1}{6}$th of a pound per yard per annum; but I have since seen it stated, in a letter from Mr. Dixon to Mr. Bidder, at $\frac{1}{10}$th of a pound per yard per annum. This was determined by taking up three rails, having them well cleaned and weighed, and then putting them in their places, and afterwards washing and reweighing them at the end of a twelvemonth, when two of them were found to have lost $\frac{1}{2}$ lb. in weight for the 5 yards length, and the the third $\frac{3}{4}$ lb., which last was taken up from a particular situation where it was more exposed to friction. But even this does not prove that the whole loss of weight is in the upper face of the rail; and if it did, it would be, as I have

before observed, a stronger reason for not turning the rail: and, on the other hand, should the waste not be on the upper surface, the provision alluded to is unnecessary. Mr. Rastrick informs me, that even the small fins left at the meeting of the rolls are still quite distinctly seen on the face of the upper table. And Mr. Stephenson states, that the marks of the tools left in turning the flanches of the wheels are seldom obliterated; which proves, at all events, that there is no side wear.

Mr. George Bidder, who attributes all the waste to the wear on the upper surface, estimates the annual reduction at $\frac{1}{90}$th part of an inch; in which case the rails would not last more than thirty years before they would require to be replaced. And it then becomes a question, whether, in point of economy, it would not be better to lay an additional third of an inch upon the upper table, which would, by this reckoning, make the rail last sixty years. This increase of $\frac{1}{3}$d of an inch would call for an additional expense, to the amount of about $7\frac{1}{2}$ per cent. on the present cost; and this $7\frac{1}{2}$ per cent, at compound interest, would amount to about 30 per cent.

in thirty years. If, therefore, a charge of 30 per cent. at the end of thirty years, would meet the amount of re-manufacture, and supply the waste, the two accounts would be about balanced. In this case, I must consider the latter as preferable. 1st. Because the other plan would increase the weight of the bar, and the difficulty of the manufacture, and probably diminish its soundness. 2d. Because thirty years' experience may introduce improvements, of which, at the end of that period, it would be desirable to take advantage. And, lastly, because I do not (judging from the opinion of different practical men,) think that it has yet been clearly determined what part of the waste is due to wear on the upper face.

To return again to the subject of the best-formed section, I beg to repeat, that whatever figure the above, or other considerations, may lead practical men to adopt in the upper or lower table and rib, it will be fully sufficient for the purposes of calculation, to consider them as rectilinear, which will greatly facilitate the investigation, without sensibly affecting the results.

Comparative Strength of differently-formed Parallel Rails.

Let A B C D (fig. p. 57) represent any rectangular rail with a bottom table; *n n* its neutral axis; *c* the centre of compression, *c n* being $\frac{2}{3}$ of *h n*. Now, the tension of each fibre being as its distance from the neutral axis, and that of the lower fibre being given equal to *t*, the tension at any variable distance *x* will be $\frac{t\,x}{d}$ (*d* being taken to denote the whole depth *n s*), and therefore the sum of all the tensions will be,

$$\frac{t}{d'} \int x.\,dx \qquad (1)$$

which, therefore, become known, *x* being taken within its proper limits, according to the figure of the section.

But as the effective resistance of each fibre is also as its depth below the line *n n*, the sum of all the resistances will be,

$$\frac{t}{d'} \int x^2.\,d\,x \qquad (2)$$

x being taken here also within its proper limits.

And then to find the centre of tension, or that point into which, if all the tensions were collected, the whole resistance would be the same as in the actual case, this would be given by the formula :

$$\frac{\int x^2.\, dx}{\int x.\, dx} \qquad (3)$$

which is precisely the expression for the centre of oscillation of a disc of the same figure.

We have hence the following general rule for finding the resistance or the weight which any given bar or rail will support at its middle point, within the limits of its elastic power, that is,

Calling the integral of formula (1) = A
Ditto ditto formula (2) = B
Ditto ditto formula (3) = D
And the distance cn = C

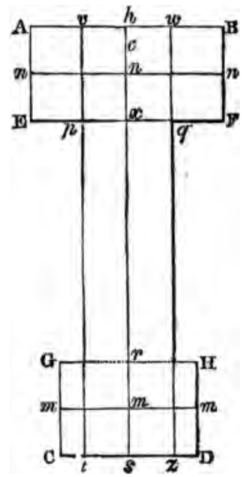
then, referring the sum of all the resistances B to the common centre of compression, we have

$$D :: D+C :: B : \frac{B\,(D+C)}{D}$$

which is the whole effect.

For those who understand the integral calculus, this solution is sufficient; but as the article will probably be consulted principally by

practical men, it will be more convenient to give a specific solution for a rail, embracing under one general figure all the usual forms, the only variations being in the depth, breadth, and thickness of the parts.



Let ABCD represent such a section, of which all the dimensions are given, as also the position of *nn* the neutral axis, the point *c* which is the centre of compression, *cn* being $\frac{2}{3}$ds of *nh*, and the point *m* which is in the centre of *rs*. The breadths *nn* and *mm* are also known. Then the resistance of the whole section referred to the common centre of compression *c*, may be considered to be made up of the three resistances.

1st. Of the middle rib, continued through the head and foot tables, *v t z w*.

2d. Of the head A E F B, minus the breadth of the centre rib.

3d. Of the lower web, G C D H, also minus the continuation of the centre rib.

Now, t being taken to represent the tension of iron per square inch, just within its limits of elasticity, we shall have

1. Resistance of... $vtzw = \frac{1}{3} hs . ns . pq . t$

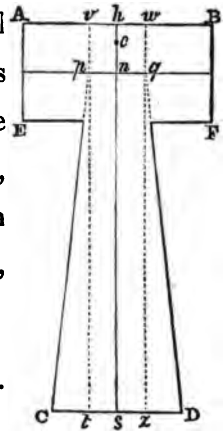
2. Resistance of AEFB $= \frac{1}{3} hx . nx . (nn - pq) \frac{nx}{ns} t$

Now, let $nm + \frac{rs^2}{12\ nm} = \delta'$, and $\delta' + cn = \delta''$, then

3. Resistance of GCDH $= nm . rs . (mm - pq) \frac{\delta''}{d'} t$

These three resistances being computed, let their sum be called s, and the clear bearing l; then $\frac{4\ s}{l} = w$, the load the bar ought to sustain at its middle point, for an indefinite time, without injury to its elasticity.

Trapezoidal Rail.



Produce the sloping sides till they intersect the neutral axis in p, q. Then the rule for the head AEFB, and middle rib, $vtzw$, will be the same as given above; and for the two sides, pCt, qDz, the formula is,

$$\frac{1}{3}\left(\frac{3}{4}ns+cn\right)\times(\mathrm{CD}-pq)\,ns^{*}t.$$

The sum of this, with that of the head and middle rib, multiplied by 4, and divided by l, as before, will be the weight required.

Another general and very curious mechanical method of finding the resistance of a railway bar, is also suggested by the remark in p. 56, viz. that the centre of tension is the same as the centre of oscillation of a disc of the form of the section,

* This includes the small dotted part of the triangular sides in the head and in the sides, but the amount is so very inconsiderably in error, as to be nearly or wholly insensible in the result.

cut off at its neutral axis, which in words may be given as follows :—

Find the centre of oscillation, and the centre of gravity of the area below the axis, by the usual mechanical methods, and call the distance of the former below the neutral axis o, that of the latter g, the area a, the depth d', and the distance $cn=c$, the tension t, and bearing l as usual, then the weight the bar will support will be,

$$w=4\,\frac{(o+c)\,agt}{ld'}$$

The preceding rules, however, will be generally more convenient, particularly when some of the dimensions become fixed, as necessarily happens in such cases as we are considering. For instance, whatever figure may be given to the transverse section, the head may generally be supposed to occupy $\frac{2}{5}$ths of it; and therefore, in the larger rails, to have about two inches section, and to be one inch deep, that the lower web, when there is one, is the same depth as the head, and that the neutral line bisect the head,

or upper table.* With these as fixed dimensions, the preceding formula, page 58, are reduceable to words at length. They apply, however, only to the larger rails; for other cases, it will be best to have recourse to the general rules.

Resistance of the Head or Upper Table.

1. Subtract the thickness of the middle rib from 2 inches, and multiply the remainder by 10.

2. Subtract $\frac{1}{2}$ an inch from the whole depth, and multiply the remainder by 12.

* We have the means of computing the position of the neutral line by the data obtained from the experiments, p. 42, which show, that in rectangular bars the area is divided in the ratio of 1 to 4, or the area into the distance of the centre of gravity of the two parts as 1 to 4^2. But in inquiries of this kind, the less we have to depend on theory the better. I have, therefore, deduced the above position from the experiments on actual Railway bars, p. 70, by considering the distance nh as unknown, and equating the formula in this shape with the mean elastic strength, which is found to be $8\frac{1}{4}$ tons. The equation is, therefore,

$$\frac{1}{3}\left\{5(5-x)\,9+\frac{11.\,(1-x)^2}{5-x}\right\}=\frac{8\frac{1}{4}\times 33}{4}$$

Whence we find $x = \cdot 47$, which may be considered as $\cdot 5$, without sensible error.

Then the former product divided by the latter will be the resistance in tons due to the head, not including the continuation of the middle rib.

Resistance of the Centre Rib.

Multiply the whole depth of the rail by the whole depth, minus $\frac{1}{2}$ an inch, and that product by 10 times the thickness of the rib ; and the last product divided by 3, will be the resistance in tons of the middle rib continued through the whole depth, *i.e.* through the upper and lower tables.

Resistance of the Lower Web.

1. Multiply the whole depth of the rail, minus 1 inch, by the thickness of the bottom web, minus the thickness of the rib, and that product by 10.

2. From the whole depth of the rail subtract 1 inch, and to 12 times the square of the remainder add 6 times the remainder, and call this the first number. From this subtract twice the remainder and add 1, and call this the second

number. Then say, as the first number is to the second, so is the product obtained in the former part of the rule to the resistance of the lower web, not including the continuation of the middle rib.

Lastly, the sum of these three resistances multiplied by 4, and divided by the clear bearing length, will be the weight the rail will sustain without injury. A few examples worked at length are given below, to illustrate the rules.

Examples.

(1) In Mr. Stephenson's rail the greatest depth is 5 inches, with a plain rib, whose thickness is ·9 of an inch. Here,

$$\text{Resistance of Head} \left\{ \begin{matrix} (2-\cdot 9)\times 10 = 11 \\ (5-\tfrac{1}{2})\times 12 = 54 \end{matrix} \right\} \frac{11}{54} = \quad 0\cdot 20$$

$$\text{Ditto of Rib} \ldots\ldots\ldots \frac{4\tfrac{1}{2}\times 5 \times \cdot 9 \times 10}{3} \qquad = 67\cdot 50$$

$$\overline{\quad 67\cdot 7 \quad}$$

And $\dfrac{4\times 67\cdot 7}{33} = 8\cdot 21$ tons, the greatest weight.

Deflection with this weight $\frac{\cdot 22}{4\cdot 5} \times \frac{4^*}{3} = \cdot 066$

(2) Parallel rail of the same depth and weight, viz. 50 lbs. per yard. Here the thickness of centre rib = ·78. Hence,

$$\text{Resistance of Head.} \left\{\begin{matrix}(2-\cdot 78)\times 10=12\cdot 2\\(5-\frac{1}{2})\times 12=54\end{matrix}\right\} \frac{12\cdot 2}{54} = \overset{\text{Tons}}{0\cdot 225}$$

$$\text{Ditto Rib......} \frac{4\frac{1}{2}\times 5\times\cdot 78\times 10}{3} \text{......} =58\cdot 5$$

$$\overline{58\cdot 725}$$

And $\frac{4 \times 58\cdot 725}{33} = 7\cdot 11$ tons, the greatest weight.

Deflection with this weight $\frac{\cdot 22}{4\cdot 5} = \cdot 048$

(3) Parallel rail with bottom web, the depth being still 5 inches, the thickness of rib ·6 of an inch thickness, or breadth of section of lower web 1·32, the weight being 50 lbs.

$$\text{Resistance of Head} \left\{\begin{matrix}(2-\cdot 6)\times 10=14\\(5-\frac{1}{2})\times 12=54\end{matrix}\right\} \frac{14}{54} = \overset{\text{Tons}}{0\cdot 26}$$

$$\text{Ditto of Rib ....} \frac{4\frac{1}{2}\times 5\times\cdot 6\times 10}{3} =45\cdot 00$$

$$\text{Lower Web} \left\{\begin{matrix}(5-1)\times\cdot 72\times 10=28\cdot 8\\12(5-1)^2+24=216=\text{1st No.}\\216-7=209=\text{2d No.}\end{matrix}\right.$$

As 216 : 209 : : 28·8 : 27·94 .... 27·94

$$\overline{73\cdot 20}$$

* See pp. 19 and 47.

And $\frac{73 \cdot 20 \times 4}{33} = 8\frac{3}{4}$ tons the greatest weight.

Deflection with this weight $\frac{\cdot 22}{4 \cdot 5} = \cdot 048$.

(4) As another example, let us take a parallel rail of 50lbs per yard, depth $4\frac{1}{2}$ inches, thickness of rib $\frac{7}{10}$th of an inch, and of the bottom web 1·39.

		Tons.
Resistance of head	$\left\{ \begin{array}{l} (2-\cdot 7) \times 10 = 13 \\ (4\frac{1}{2}-\frac{1}{2}) \times 12 = 48 \end{array} \right\} \frac{13}{48} =$	0·27
Ditto of Rib . . .	$\frac{4 \times 4\frac{1}{2} \times \cdot 7 \times 10}{3}$	=42·00
Do. of lower Web . .	$\left\{ \begin{array}{l} 3\frac{1}{2} \times (1 \cdot 39 - \cdot 7) \times 10 = 24 \cdot 15 \\ 12\,(3\frac{1}{2})^2 + 21 = 168 = \text{1st No.} \\ 168 - 6 = 162 = \text{2d No.} \end{array} \right.$	
	As 168 : 162 : : 24·15 : 23·28	=23·28
		65·55

$\frac{4 \times 65 \cdot 55}{33} = 8$ tons, nearly the greatest weight.

Deflections with this weight $\frac{\cdot 22}{4} = \cdot 055$.

Remarks on the above Results.

It appears from these results, that it is always possible to produce a parallel rail of good practical proportions, which shall be as strong as

a Fish-bellied Rail of the same weight; and this being the case, I am decidedly convinced, after hearing and well weighing every argument that has been advanced in favour of the latter form, that the parallel rail is the best.

First.—Because, although it is not stronger nor stiffer in the middle point than the Fish-bellied Rail, it is both stronger and stiffer in a very sensible degree in every other point.

Second.—Its deflection of a parallel rail during the passage of a load is less every where than in the middle, which is not the case in the Fish-bellied Rail. The rise and fall of the carriage, therefore, after passing over a support, is more rapid in one case than in the other; and to this, rather than to a want of equable strength throughout, I am disposed to attribute the many failures of Fish-bellied Rails within a short distance of their point of support. There is, however, or has been hitherto, an actual want of equable strength towards the point of support in rails of this form, which cannot fail to have facilitated these fractures; but which Mr. Stevenson, by a judicious and scientific distribution of the metal,

has avoided, and no doubt such fractures would be with his rail less common; but the objection I offer above, applies not merely to the Fish-bellied Rail, but to the truly elliptical form itself if it were possible to arrive at it.

Thirdly.—The parallel rail is the best, because it enables the engineer to keep the blocks and chairs of the two rails directly opposite to each other, so that the wheels of the carriage shall pass over both supports at the same time,—a point, I believe, not hitherto much attended to, but which is, I conceive, of great importance. There can be no doubt the motion of a locomotive-carriage consists of a succession of ascents and descents; and it must be evident how much easier and better the motion would be, to have the opposite wheels both rising and both falling together, than to have one always rising while the other is falling, and the contrary. The difference is similar to that of a vessel keeping her head to the waves, and crossing their direction obliquely. And every one who has never been further than Margate must have experienced this difference.

It may be observed that the waves of the railway, or the deflections of the rails, are very small; but I would observe also, that the weights and velocities of the carriages are very great, and that it is very desirable every possible cause of momentum should be removed, particularly when it is as easy to do it as not to do it, as is the case with parallel rails, because they can always be cut to determinate lengths, but which cannot be done in the Fish-bellied Rail, in consequence of the occasional slipping of the bar in the rolls. At all events, their length cannot be varied at pleasure, which the former will admit of, and which is necessary, in going round sweeps, to preserve the blocks always parallel. For example, in going round a sweep of 800 feet, to keep the supports parallel, the rails of the inner course require to be about an inch shorter than the outer ones, and they are as easily cut into lengths of fourteen feet eleven inches as of fifteen feet, which is not practicable in the other form.

The above is my decided conviction relative to the longitudinal figure of the rails. I entered upon the inquiry without prejudice, I felt sen-

sible of the honour which the General Meeting had done me in confiding the question to my investigation; and I have given to it (after obtaining the requisite data,) all the attention necessary to arrive at a certain conclusion.

The following experiments were made on different rails, and the results may be compared with the preceding calculations.

Experiments on the Resistance and Deflection of Rail-way Bars.

Mr. Stephenson's Fish-bellied Rail, 50 lbs. per Yard.

Bar No. 1.			Bar No. 2.		
Weights.	Deflections by Index.	Deflections for each Ton.	Weights.	Deflections by Index.	Deflections for each Ton.
1	·035		1	·014	
2	·045	·010	2	·022	·008
3	·055	·010	3	·030	·008
4	·065	·010	4	·042	·012
5	·071	·006	5	·050	·008
6	·076	·005	6	·062	·012
7	·087	·011	7	·075	·013
7½	·095	·016]	8	·085	·010
			9*	·101	·016
			10	* Elasticity injured.	
			11	·300	

Bar No. 3.			Bar No. 4.		
Weights.	Deflections by Index.	Deflections for each Ton.	Weights.	Deflections by Index.	Deflections for each Ton.
1	·018		1	·045	
2	·025	·007	2	·056	·011
3	·038	·013	3	·065	·009
4	·054	·016	4	·075	·010
5	·062	·008	5	·084	·009
6	·069	·007	6	·095	·011
7	·080	·011	7	·105	·010
8	·094	·014	8	·110	·005
8½	·100	·012	9	·116	·006
9*	·112	·018	10	·125	·009
9½	·118	·018	11	·165	
10	·126	·014			
11	·160	·034			
17	destroyed				

Mean Deflection per Ton, Bar	No. 1. . .	·0097
	No. 2. . .	·0101
	No. 3. . .	·0110
	No. 4. . .	·0090
	Mean . . .	·0100

TABLE *continued.*

Bar No. 5, Fish-bellied. Great depth, 5 inch. Less ditto, 3¾. Thickness of Rib, $\frac{9}{10}$. Head estimated, 2 by 1.			Bar No. 6, Fish-bellied. Great depth, 3¼ inch. Less ditto, 2¼. Thickness of Rib, $\frac{7}{10}$. Head estimated, 2 by ¾.			Bar No. 7, Fish-bellied. Great depth, 3 inch. Less ditto, 2. Thickness of Rib, $\frac{6}{10}$. Head estimated, 2 by ½.		
Weight in Tons.	Deflection by Index.	Deflection for each Ton.	Weight in Tons.	Deflection by Index.	Deflection for each half Ton.	Weight in Tons.	Deflection by Index.	Deflection for each half Ton.
1	·030		0·5	·120		0·5	·033	
2	·260		1·0	·140	·020	1·0	·060 Readjusted	·027
3	·270	·010	1·5	·170	·030	1·5	·062	
4	·290	·020	2·0	·180	·010	2·0	·090	·028
5	·300	·010	2·5	·200	·020	2·5	·120	·030
6	·320	·020	3·0	·230	·030	3·0	·155	·035
7	·335	·015	3·5	·280	·050	3·5	·240	
8	·410	·060	4·0	·420	·140	4 0		
Mean deflection per Ton to 7 Tons............		·015	Mean deflection per half Ton to 3 Tons......		·022	Mean deflection per half Ton to 2 Tons......		·030
Ditto, with 7½ Tons		·107	Ditto, with 3 Tons		·066	Ditto, with 2 Tons		·060

Comparison of the above Results, with the Formulæ, page 58.—viz.

$$\text{Rib} \ldots\ldots\ldots\ldots \frac{1}{3} hs \,.\, ns \,.\, pq \,.\, t$$

$$\text{Head} \ldots\ldots\ldots \frac{1}{3} hx \,.\, \overline{nx}^2 \cdot \frac{nn - pq}{ns} t$$

Bar, No. 5.

$$\text{Here} \ldots\ldots \begin{cases} hs = 5,\ ns = 4{\cdot}5,\ pq = {\cdot}9,\ t = 10 \\ hx = 1,\ nx = {\cdot}5,\ nn - pq = 1{\cdot}1 \end{cases}$$

$$\text{Hence} \ldots\ldots \frac{1}{3} \times 5 \times 4{\cdot}5 \times 9 = 67{\cdot}5$$

$$\frac{1}{3} \times 1 \times \overline{0{\cdot}5}^2 \times \frac{11}{45} = {\cdot}20$$

$$\frac{67{\cdot}7 \times 4}{33} = 8\tfrac{2}{11} \text{ tons}$$

$$\frac{{\cdot}22}{4{\cdot}5} \times \frac{4}{3} \times {\cdot}066 \text{ deflection}$$

Bar, No. 6.

$$\text{Here......}\begin{cases} hs = 3{\cdot}25,\ ns = 2{\cdot}88,\ pq\ {\cdot}7,\ t = 10 \\ hx = {\cdot}75,\ nx = {\cdot}375, nn - pq = 1{\cdot}3 \end{cases}$$

$$\text{Hence ...}\ \frac{1}{3} \times 3{\cdot}25 \times 2{\cdot}88 \times 7 = 21{\cdot}84$$

$$\frac{1}{3} \times {\cdot}75 \times \overline{{\cdot}375}^2 \times \frac{13}{2{\cdot}88} = \quad {\cdot}15$$

$$\overline{21{\cdot}99} = s$$

$$\frac{4\,s}{33} = 2\tfrac{2}{3} \text{ tons}$$

$$\frac{{\cdot}22}{2{\cdot}88} \times \frac{4}{3} = {\cdot}092 \text{ deflection}$$

Bar, No. 7.

$$\text{Here......}\begin{cases} hs = 3,\ ns = 2{\cdot}75,\ pq = {\cdot}6,\ t = 10 \\ hx = {\cdot}5,\ nx = {\cdot}25,\ nn - pq = 1{\cdot}4 \end{cases}$$

$$\text{Hence ...}\ \frac{1}{3} \times 3 \times 2{\cdot}75 \times 6 = 16{\cdot}50$$

$$\frac{1}{3} \times {\cdot}5 \times \overline{{\cdot}25}^2 \times \frac{14}{275} = \quad {\cdot}05$$

$$\overline{16{\cdot}55} = s$$

$$\frac{4\,s}{33} = 2{\cdot}06 \text{ tons}$$

$$\frac{{\cdot}22}{2{\cdot}75} \times \frac{4}{3} = {\cdot}106 \text{ deflection.}$$

NOTES AND ILLUSTRATIONS.

In order to avoid embarrassing the detail of the experiments with mathematical solutions, I have generally only stated the equations and their results in the preceding paper; but as, in its present form, some persons may wish to see the solutions themselves, I shall add here such as involve any difficulty, or which require any illustration.

The first which occurs is the integration of the differential—

$$\frac{x^2 dx}{\frac{d^3}{l^3}(2\,lx - x^2)^{\frac{3}{2}}}$$

This may be put under the form,

$$\frac{l^3}{d^3}\int x^2\,(2\,lx - x^2)^{-\frac{3}{2}} dx$$

Or making $2\,l = p$ under the form,

$$\frac{l^3}{d^3}\int x^{\frac{1}{2}}\,(p - x)^{-\frac{3}{2}}$$

Now the part under the integrating sign in a series, becomes,

$$\int \frac{x^{\frac{1}{2}}\,dx}{p^{\frac{3}{2}}} = \frac{2}{3}\frac{x^{\frac{3}{2}}}{p^{\frac{3}{2}}}$$

$$\int \frac{3\,x^{\frac{3}{2}}\,dx}{2\,p^{\frac{5}{2}}} = \frac{2}{5}\cdot\frac{3\,x^{\frac{5}{2}}}{2\,p^{\frac{5}{2}}}$$

$$\int \frac{5\,.\,3\,x^{\frac{5}{2}}\,dx}{4\,.\,2\,p^{\frac{7}{2}}} = \frac{2}{7}\cdot\frac{5\,.\,3\,x^{\frac{7}{2}}}{4\,.\,2\,p^{\frac{7}{2}}}$$

$$\int \frac{7\,.\,5\,.\,3\,x^{\frac{7}{2}}\,.\,dx}{6\,.\,4\,.\,2\,p^{\frac{9}{2}}} = \frac{2}{9}\cdot\frac{7\,.\,5\,.\,3\,x^{\frac{9}{2}}}{6\,.\,4\,.\,2\,p^{\frac{9}{2}}}$$

&c. &c.

Which when $x=\frac{1}{2}p=l$, may be written,

$$\frac{1}{\sqrt{2}}\left\{\frac{1}{3}\cdot\frac{1}{1}\right. = \cdot 3333$$

$$+\frac{1}{5\,.\,2}\cdot\frac{3}{2} = \cdot 15000$$

$$+\frac{1}{7\,.\,2^2}\cdot\frac{5\,.\,3}{4\,.\,2} = \cdot 06695$$

$$+\frac{1}{9\,.\,2^3}\cdot\frac{7\,.\,5\,.\,3}{6\,.\,4\,.\,2} = \cdot 03040$$

$+$ &c. $=$ &c.

This series, after a few terms, may be considered nearly equivalent to a geometrical series,

having the ratio $\frac{1}{2}$, and may be summed accordingly. We have thus ultimately for the original expression,

$$\frac{l^3}{d^3} \times \frac{1}{\sqrt{2}} \times \cdot 6095, \&c. = \cdot 41 \frac{l^3}{d^3},$$

as given in p. 19.

Note to p. 52.

It may be convenient to show the origin of these formula, particularly the third, which is not investigated in the preceding pages, except that it has been shown generally, that if d' denote the depth of the lower fibre below nn, and its tension be made $= t$, and any variable distance $= x$,

That $\frac{t}{d'}\int x\,dx =$ sum of all the tensions to a unit of breadth.

That $\frac{t}{d'}\int x^2 dx =$ sum of all the resistance referred to the axis n.

And $\dfrac{\frac{t}{d'}\int x^2\,dx}{\frac{t}{d'}\int x\,dx} = \delta'$ distance of centre of tension.

From which it follows,

that $\frac{t\delta'}{d'}\int x \, . \, dx$ = sum of all the resistances for a unit of breadth, x being taken in its ultimate state.

Now, in the rib, when $x = d'$, $\delta' = \frac{2}{3}$ d, and

$\int x dx = \frac{1}{2}$ d'^2, whence the above becomes

$$\frac{1}{3} d'^2 \, t;$$

but to refer this to the centre of compression c, we have (calling the whole depth d)—

$$\frac{2}{3} d' \; : \; \frac{2}{3} d \; :: \; \frac{1}{3} d'^2 t \; : \; \frac{1}{3} d d' t;$$

and introducing the breadth pq, it becomes—

$$\frac{1}{3} hs \, . \, ns \, . \, pq \, . \, t.$$

In the same way, calling the tension at $x = t'$, and the breadth $(nn - pq)$, we have for the resistance of the head—

$$\frac{1}{3} hx \, . \, nx \, . \, (nn - pq) \, t';$$

but the tension at $x = \frac{nx}{ns} t$;

therefore, substituting this for t', we have

$$\frac{1}{3} hx \, . \, \overline{nx^2} \, \frac{(nn - pq)}{ns}.$$

For the lower web—

$$\frac{\frac{t}{d'}\int x^2\,dx}{\frac{t}{d'}\int x\,d^x} = \delta'$$

Calling $nr = d''$, and x any variable distance below r, it becomes—

$$\frac{\int (d'' + x)^2\,dx}{\int (d'' + x)\;dx} = \delta';$$

which, when $x = rs$, gives

$$\delta' = nm + \frac{rs^2}{1{\cdot}2\;mn}$$

and $\frac{t}{d'}\int (d'' + x)\,dx = \frac{t\delta'}{d'}\,nm\,.\,rs$

whence the resistance referred to nn is, for the breadth $(mm - pq)$

$$nm\,.\,rs\,(mm - pq)\,\frac{t\delta'}{d'};$$

and calling $\delta' + nc = \delta''$, it is, when referred to c,

$$nm\,.\,rs\,(mm - pq)\,\frac{\delta'' t}{d'}$$

which is the formula in question.

In a similar way, the formula for the trapezoidal rail is obtained.

NOTE to p. 61.

Another equation, on which it may be well to offer a few remarks, is that given in p. 61, viz.

$$\frac{1}{3}\left\{5(5-x)\,9+\frac{11.(1-x)^2}{(5-x)}\right\}=\frac{8\frac{1}{4}\times 33}{4}$$

This is drawn from the general solution, p. 58, viz.

$$\left.\begin{array}{l}\frac{1}{3}hs\,.\,ns\,.\,pq\,.\,t\\[6pt] +\frac{1}{3}hx\,.\,\overline{nx}^2.\,\frac{nn-pq}{ns}\,t\end{array}\right\}=s$$

and $\frac{4\,s}{l}=w.$

Taking the result of the experiments, p. 70, at $w=8\frac{1}{4}$, and the dimensions of that bar as known quantities, every thing in the above is given except the position of the line nn. Calling, therefore, $hn=x$, and substituting the proper numerical values for the other parts, we have,

$$\frac{1}{3}.\,5\,.\,(5-x)\,.\,9+\frac{1}{3}\,(1-x)^2\,.\,\frac{1\cdot 1}{5-x}\,.\,10=\frac{33\,.\,8\frac{1}{4}}{4}$$

This reduces first to—

$$45\,(5-x)^2+11\,(1-x)^2=204\cdot 18\,(5-x)$$

then to $x^2-\frac{267\cdot 82}{56}x=-\frac{115\cdot 1}{56}$

whence the value of $x=\cdot 484$.

Here t is taken at ten tons, according to our first mean results; but if instead of this we consider it like x as an unknown quantity, the equation is,

$$4{\cdot}5\, t\, (5-x)^2 + 1{\cdot}1\, t\, (1-x^2) = 204{\cdot}18\, (5-x.)$$

that is, t and x are dependent quantities, and every change in the value of t produces a corresponding change in the value of x.

If $t = 10{\cdot}5$, then the equation is,

$$45\, (5-x)^2 + 11\, (1-x)^2 = 194{\cdot}54\, (5-x).$$

Whence $x = {\cdot}736$.

Again, we may find x quite independently of these considerations, by taking the ratio of the surfaces of tension and compression found in p. 42, viz. 1 : 4; and these into the distances of their respective centres of gravity; or, which is the same, the whole quantity of compression to that of extension as 1 to 4^2.

Considering this as a general law, and dividing our area accordingly, we have,

$$16\, x^2 = (1-x)^2 + 3{\cdot}6\, (3-x), \text{ or,}$$

$$16\, x^2 + 5{\cdot}6 x = 11{\cdot}8$$

from which we find $x = {\cdot}720$.

Hence it appears that whatever method is pursued, the resulting numbers are exceedingly approximative. It has, however, been thought best for the object in view, to derive our final data from that case most resembling the actual subject of inquiry,—which is that of Railway

Bars having necessarily an upper table; and in these, t being taken as equal to ten tons in good iron, the neutral line may be considered to divide the area of the upper table into two equal parts; and on these are founded the rules given in p. 62. In other cases it will be better to determine x, as in the last case, and proceed by the general rule.

I know that it has been advanced, on theoretical principles, that at the commencement of strain the neutral axis is in the centre of gravity of the area of section, but this consideration does not enter into my investigation. I have not examined the question on theoretical, but on mechanical principles, with a view to one specific object, and have purposely avoided resting any point on mere hypothesis. Every thing is made to depend on experimental results; and, from the uniformity and agreement of these, I have every confidence the rules founded on them will enable practical men to compute such cases as may occur with all the precision that can be desired.

As another example, let it be proposed to find the strength and deflection of Mr. Stephenson's rail inverted.

Call the distance of the neutral axis from the top, $hn = x$. (Fig. p. 57.)

Then $\quad \cdot 9\,x =$ area of compression.

$\frac{1}{2}\,x =$ distance of centre of gravity.

$\overline{\cdot 45\,x^2} =$ quantity of compression.

Again $\quad \cdot 9\,(5-x) = \left\{\text{area of extension of the middle rib.}\right.$

$\frac{1}{2}\,(5-x) =$ distance of centre of gravity.

$\overline{\cdot 45\,(5-x)^2} = \left\{\text{quantity of extension of middle rib.}\right.$

Also $\quad 1\cdot 1 =$ area of lower web.

$(4\cdot 5-x) =$ distance of centre of gravity.

$\overline{1\cdot 1\,(4\cdot 5-x)} = \left\{\text{quantity of extension of lower web.}\right.$

Since as compression to extension, so is $1^2 : 4^2$, we have,

$$7\cdot 2\,x^2 = \cdot 45\,(5-x)^2 + 1\cdot 1\,(4\cdot 5-x) \text{ or}$$

$$x^2 + \cdot 829\,x = 2\cdot 4$$

Whence $x = 1\cdot 185$, or $1\cdot 2$, nearly.

Now by the rules for the middle rib and lower web, (p. 58,) we have,

$$hs = 5,\ \ ns = 3\cdot 8,\ \ pq = \cdot 9,\ \ t = 10, \text{ and}$$

$$\frac{1}{3} hs \,.\, ns \,.\, pq \,.\, t \qquad = \qquad \overset{\text{Tons.}}{57}$$

Also $\left.\begin{aligned} nm = 3\cdot 3,\ mm - pq = 1\cdot 1,\ d' = 3\cdot 8 \\ \delta'' = nm + \frac{1}{12\,.\,\overline{mn}^2} + cn = 4\cdot 1 \end{aligned}\right\}$

Wherefore,

$$nm \,.\, rs\,(mm-pq)\,\frac{\delta''}{d'}\,t \qquad = \qquad 39\cdot 1$$

$$\overline{96\cdot 1}$$

$$\frac{96 \cdot 1 \times 4}{33} = 11\tfrac{7}{11} \text{ tons greatest weight.}$$

$$\frac{\cdot 22}{3 \cdot 8} \times \frac{4}{3} = \cdot 0772 \text{ deflection with that weight.}$$

Whence in the two positions of the rail we have,

$$8\tfrac{1}{4} : 11\tfrac{7}{11} :: 1 : 1 \cdot 4 \text{ ratio of strengths.}$$

$$3 \cdot 8 : 4 \cdot 5 :: 1 : 1 \cdot 18 \left\{ \begin{array}{l} \text{ratio of ultimate de-} \\ \text{flections.} \end{array} \right.$$

$$\frac{3 \cdot 8}{8\frac{1}{4}} : \frac{4 \cdot 5}{11 \cdot \frac{7}{11}} :: 1 : \cdot 84 \left\{ \begin{array}{l} \text{ratio of deflections with} \\ \text{equal weights.} \end{array} \right.$$

Erratum.—The reader is requested to make the upper line of the first figure, p. 22, continuous throughout.

REPORT

ADDRESSED TO

THE DIRECTORS

OF THE

LONDON AND BIRMINGHAM RAILWAY COMPANY.

GENTLEMEN,

THE accompanying paper contains the details of the experiments I have made, in conformity with the resolution of the General Meeting of the Proprietors of the London and Birmingham Railway Company, held at Birmingham, on the 13th of February, and I am in hopes several important data and rules have been thus elicited. These will be found in the paper referred to; but it may be convenient to state the results in this place, referring to the proper pages for the experiments and investigation on which they are founded.

It has been ascertained (page 27), that a malleable iron bar of any length is extended $\frac{1}{10000}$th part of its length by a direct strain of a ton per inch on its sectional area; and that, when strained with ten tons per inch, or when stretched $\frac{1}{1000}$th part of its length, its elasticity is injured, and the bar will not return to its original state.

Now, as the contraction of iron, between summer and winter, amounts to $\frac{1}{2000}$th part of its length, it follows, that the bars cannot be fixed permanently to the chairs and blocks, without great danger of drawing so much upon their strength, as materially to impair their efficiency for bearing a great passing load.

It follows also as a consequence, that if the rails and chairs must not be permanently fixed to each other by direct means, it ought not to be attempted by indirect means, viz. by cotters, keys, or wedges; for, either these will hold the rail to the chair, or they will not. If they do hold fast, they produce all the mischief which permanent fixing would occasion; and if they draw, then they do no good, although they may

still do mischief. Whence I am led to conclude, that the rails should have no greater attachment to the chairs than is sufficient for preserving them steady while the load is passing.

My next experiments were directed to finding the position of the neutral axis in malleable iron, for without this datum, the strength of rails, of differently formed transverse sections, cannot be computed and compared with each other, at least, without the expensive mode of having them first made, and then their strengths found by experiment. In this inquiry, as in the preceding, I have succeeded to my entire satisfaction; and, by the results obtained, have formed rules of very simple character, which will enable any person to compute with great precision the bearing strength of a bar of any proposed section within the limits of its elastic or restoring power, and also the amount of the deflection it will sustain under this or any lesser load. I have demonstrated by these means, that we may find certain practicable forms of parallel rails, which shall be, weight for weight, equally as strong as the fish-bellied rail, when loaded at their

middle point, and of course stronger in every other part. For which reasons, and for others I have explained in page 66, I feel fully convinced that the parallel rail formed according to the requisite proportions, is decidedly the best.

Such are the results of my experimental researches on this subject; and here, perhaps, I ought to close my report, leaving it to practical men to work out the conditions I have shown to be necessary; but I hope I may be allowed to offer some suggestions on a few practical points,—a task for which I feel myself the better qualified, by being a week associated with Messrs. Rastrick and Wood, in examining the models sent in for the prize, and thus benefiting by their practical skill and remarks. To which I may also add, the advantage I derived from examining so many models, several of them exceedingly ingenious, and accompanied with descriptions, containing very sensible remarks on different modes of practice.

In the first place, as I have already stated, I am decidedly convinced that the rail should be parallel; that its whole depth should not be

less than $4\frac{1}{2}$ or $4\frac{1}{4}$ inches; that the thickness of the middle rib should not exceed that—which is essential to the perfect manufacture of the bar; and that the lower web (without any reference to the distant contingency and dangerous proposition of turning the rail,) should be made of the best form for present purposes, viz. for giving to it a steady bearing in its seat.

With respect to the joint chair, I do not think any better form can be devised, than *the whole* chair proposed by Mr. Daglish, viz. in which he uses no filling-up piece, but with a different wedge. It is my opinion, that by well-gauging both the ends of the rails and chairs, and then leaving the former free, we should best comply with the conditions I have endeavoured to show to be desirable, if not absolutely necessary.

To carry this into practice, however, so as to enable the rail to be removed if necessary, it is essential that the pin, which holds the chair to the block, should be allowed to fall down into the stone, for which purpose the lewis-pin, proposed by Mr. Swinbourn, is well suited; but still I think that, combining this ingenious idea of

fixing, with that proposed by Mr. Vignoles, a better effect might be expected; that is, instead of the lewis, I should recommend the large-headed bolt, with or without a loose washer, to cut a hole in the back of the block to the depth of about two inches, up which the head of the bolt may be passed, which would allow it to be dropped down when necessary, and admit of application in a more simple, and probably in a more effectual manner, than the lewis, if, as I have been informed, the latter is liable to split the block.

For the intermediate chairs, I think a slight modification of Mr. Stephenson's would best answer the purpose; that is, I would support the rail in the chair simply by the ends of two plain-ended pins, so as to give it the requisite steadiness with as little friction as possible. Of course, I would have these pins pointing horizontally, or upwards, instead of downwards, as they do in the chair in question.* I do not

* It may be worth consideration, whether, if this mode of fixing were adopted, it would not be practicable and advantageous to introduce pieces of felt, or other substance, within

conceive such pins would be necessary in the joint chair, but provision might be made for them, and they could be applied, if necessary.

I have no doubt, practical men, who have taken a different view of this subject, and have thought it necessary to fix every thing as fast as possible, will see objections to the light fixing I have proposed, but without attempting to anticipate and answer those objections, I can only say, after having, I believe, heard every thing that can be advanced on the subject, that my opinion is such as I have stated.

I have, above, alluded to the guaging the ends of the rails and the openings in the joint chair, and I have also spoken in the description of my experiments (p. 67), of the advantage of keeping the blocks of the two lines of rails parallel. On all these points it is probable I shall be considered by many as entering into refinements neither called for nor practicable, in the case of railways; but I would ask, why is it found that so much breakage takes place,

the seat of the chair, which would greatly subdue the jars that take place between metal and metal.

and that so many repairs are rendered necessary? There is no theoretical reason why a heavy load, passing with great velocity, should cause more damage than the same load passing slowly, if the road were perfect; the mischief, therefore, is in the imperfect practical execution, and the disregarding small inequalities, as we would disregard them in common cases. It has perhaps never occurred to such persons, that a difference of level at a joint chair, between the two abutting rails of only $\frac{1}{10}$th of an inch, will, when the carriage is moving from the higher to the lower level at its greatest speed, cause the wheel to pass the distance of a foot without pressing on the rail, and consequently throwing the whole weight, which ought to be borne equally by the two rails, wholly upon one; yet this is a fact resting on a natural law, and cannot be otherwise. To fall $\frac{1}{10}$th of an inch by the action of gravity requires $\frac{1}{44}$th part of a second, and in that time the carriage will have advanced a foot; consequently, for that space the whole weight has been borne by one rail only. It may be said there are springs provided,

which assist gravity to bring down the wheel. I am afraid, however, after allowing for their inertia, that such aids are very inefficient; at all events, they furnish no arguments against having every thing as accurate as possible. Again, with reference to the abutting rails, I was certainly surprised when a gentleman officially attached to the Manchester and Liverpool Railway, informed me, that in some parts of their line, the rails were half an inch apart, and that it was not thought injurious. But why, I would ask, whether injurious or not, have them half an inch apart, when they never need be open above $\frac{1}{10}$th of an inch; and, for more than half the year, not above $\frac{1}{20}$th of an inch, if proper care be taken in laying them down? Hitherto an idea has prevailed, that in laying down the rail, $\frac{1}{8}$th or $\frac{1}{10}$th of an inch must be left for expansion, and whether hot weather or cold, the same allowance is made; consequently, if the rail is laid in the summer, the $\frac{1}{8}$th of an inch becomes nearly a quarter of an inch in the winter, provided the contraction takes place in the same direction in two adjacent rails;

but if in a contrary direction, it becomes half an inch, or nearly so, as my informant states the fact occasionally to be. To prevent this, I would, as stated at p. 31 of my experiments, have each rail fixed to one chair, and to one chair only; and I would have three steel plates, the thicknesses of the proper spaces, to be left between the rails, according to the temperature—one between 15° and 35°, another between 35° and 65°, and another for all temperatures above 65°, whereby to regulate the distances of the rails. This, again, will, I have little doubt, be considered an unnecessary refinement; but to such objections I reply, that this acuracy costs nothing additional in the execution, and may therefore, at all events, be as well attended to as not.

It only remains now for me to make a few observations upon the absolute requisite strength of the bars, and the tests of strength to which they ought to be submitted, before they are reported, and received as efficient.

First.—With respect to their absolute strength, the amount of this will depend upon the weight of the locomotive intended to be employed,

which I shall here assume at 12 tons; and, notwithstanding six wheels may be used, I shall, for the sake of being on the safe side, consider only four, or that each wheel bears one-fourth of the whole weight, or three tons. I will also suppose that, whether my suggestions are acted on or not, cases may occasionally occur, when the weight, which ought to be borne equably on two rails, is, from irregularities in the road, thrown all on one. This gives the greatest bearing load 6 tons, and I would then have a surplus strength of 50 per cent., making 9 tons; that is, I would have rails whose absolute elastic or restoring power should be 9 tons, and I would test every rail to 7½ or 8 tons. Such a test would be perfectly harmless on bars of good iron; and, unless it is carried thus high, it is impossible to detect bars made of an inferior quality, which show more stiffness in the commencement than the best iron: but their elastic power at length yields suddenly, and the bar becomes useless. Such iron should, of course, be excluded, unless indeed it be contracted for, and the rails proportioned accord-

ingly. This testing should be carried on in the line of works by a proper person, and the manufacturer left free to use his own plans without superintendence; as practised by the Admiralty in the receipt of their iron cables. There can be no doubt, that if the cables were sent to sea without proof, and every failure of a link attributed to a want of sufficient dimensions, before this time we should have had cables for the several rates of vessels of much larger bolts than at present, thereby adding, at a great expense, much unnecessary and even injurious weight, as appears to be the tendency of the present practice in railway bars.

The proof I would recommend is as follows:—On the line, near the place where the work is going on, all the intermediate chairs, in one length of rail should be removed, and over this space the bar for trial should be placed. A carriage then, rightly adjusted for weight, and with wheels at a proper distance to bring the requisite strain on the metal, should be passed over twice; when, if no permanent deflection be observed, the bar is to be considered sound,

and removed, and its place supplied by another, to undergo the same test. In this way I consider that 50 or 60 bars a day may be tested at a very trifling expense, but it should be done under the superintendence of a person on whose report reliance may be placed, and to whom the other minutia I have mentioned might also be entrusted.

I mention this, because, if the plan be acted upon at all, it should be followed up strictly, as well in justice to the Company and Iron-Master, as to the proposition itself.

When the laying down has proceeded a certain distance, the chairs may be replaced, and those of another rail removed, to form a new testing ground nearer to the point of active operations.

I have considered the other suggestion for testing by percussion, but do not think it would be recommendable.

For the guaging, I would recommend an over and under guage, according to the plan followed by the Ordnance Board, in the receipt of shot and shell.

I think it is possible, by a slight modification of the form of rail I have comprised in my fourth example (p. 65), to give to it a strength of 9 tons, without any increase of weight. I have allowed rather more metal for the head, I believe, than is generally employed, which, if transferred to the lower web, would give all the additional strength required; or, perhaps, the centre rib might bear a slight reduction. At all events, leaving every thing as it is, except adding 2lbs. per yard to the bottom web, the rail would come up to the whole strength of 9 tons, as required. And here, I would observe, is the great advantage of working by rule rather than by opinion, for if we had only the latter to guide us, we should be hard to believe that an increase of $\frac{1}{25}$th in the weight could be made to add about $\frac{1}{9}$th to the strength and stiffness of the bar; yet such is unquestionably the case.

In conclusion, I feel it my duty to state, that through the liberal permission of my Lords Commissioners of the Admiralty, I have had every convenience I could desire for conducting the

experiments; that the London Committee have caused to be supplied every instrument and material I had occasion for; and that I have been much indebted to Messrs. Lloyd and Kingston for their cheerful assistance, and ingenious suggestions on various occasions. On my own part, I will only say, that I entered upon the inquiry without prejudice; that I have made the best use in my power of the means placed at my disposal; have faithfully recorded every result as it was noted down at the moment of observation; and that I am in hopes the laws and rules I have deduced are legitimate, and may be found useful, by enabling practical men to compute results which they have hitherto been only able to conjecture.

I have the honour to be,

Gentlemen,

Your obedient Servant,

PETER BARLOW.

To the Directors of the
London and Birmingham Railway Company.

WOOLWICH,
March 25, 1835.